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Giambattista Giacomin: Random Polymer Models

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Random polymers are a very wide field of research, actively studied by theoretical, mathematical and computational physicists, besides biologists and chemists.

The book focuses on the mathematical aspects of this subject, and the author concentrates his attention on two particular models: *directed polymers in interaction with a defect line* and *copolymers at a selective interface*. Such models are typically used to describe heteropolymers in the vicinity of the interface between two solvents, $(1+1)$-dimensional wetting models, the interaction of flux lines with columnar defects in superconductors, and the DNA denaturation transition. Apart from their (bio)-physical motivation, they are extremely interesting objects from the point of view of theoretical and mathematical physics, and research in this field is presently very active.

As a typical example, think of a directed polymer $\{(n, S_n)\}_{n=0,1,\ldots}$ in $\mathbb{Z}^{d+1}$ (a d-dimensional simple random walk (SRW) stretched in the discrete time direction), which gets the reward ϵ_n (which may actually be a penalty, if $\epsilon_n < 0$) whenever $S_n = 0$. One may consider the case where the ϵ_n are all equal (homogeneous model), periodic (weakly inhomogeneous model) or random and i.i.d. (random model). In all three cases, there occurs a localization/delocalization transition: when most ϵ_n are positive and large the polymer sticks to the line $\{0\} \times \mathbb{Z}$ (localized phase), while if small or negative ϵ_n prevail then it behaves like a simple random walk (delocalized phase).

This example is easily generalized to the case where the returns to the origin of the SRW are replaced by a renewal process $\tau = \{\tau_0, \tau_1, \ldots\}$ whose inter-arrival distribution has a power-law tail: $\mathbf{P}(\tau_i - \tau_{i-1} = n) \sim n^{-\alpha-1}$ for some $\alpha > 0$ (in the SRW example, $\alpha = 1/2$ for $d = 1$ and $\alpha = d/2 - 1$ for $d \geq 2$).

The main focus of the book is on the disordered case. However, the homogeneous and periodic cases, which can be (rather) explicitly solved via renewal theory techniques, are also treated in detail in Chaps. 2 and 3, respectively. In particular, for the homogeneous model the transition can be of any given order, from first to infinite, varying α. This is therefore an

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ideal testing ground to study the effect of disorder on phase transitions, and this question is considered especially in Chap. 5. Another interesting question is to compute the boundary between localized and delocalized phases: while for the homogeneous model this is trivial, the same is not true for the periodic case (where the problem reduces to the computation of the Perron-Frobenius eigenvalue of a matrix whose dimension is large if the period is) and especially not for the random model, where only bounds are known (cf. Chaps. 5 and 6).

Apart from questions like the computation (or estimation) of free energy, critical point and critical exponents, a very natural question is about path properties: given that the system is, say, in the localized phase, what are the typical (with respect to the Gibbs measure and to the disorder distribution) properties of the trajectory $\{S_n\}_{n\geq 0}$, or of the set of contacts τ? This analysis is undertaken in Chaps. 7 (localized phase) and 8 (delocalized phase).

Chapter 9, where numerical simulations are discussed, may be of great practical interest for the physics audience. The author discusses algorithms which allow to compute the partition function in polynomial time (either exactly or in an approximate but controlled way (Fixman-Freire algorithm)) and, most importantly, he explains how concentration-of-measure inequalities allow to perform statistical tests of localization with an extremely small level of error.

Practically all the results presented in the book (with the notable exception of the Brownian weak-coupling limit of the random copolymer model of Sect. 6.3) are accompanied by detailed proofs. While most results have appeared in published papers before, many of them have been generalized in the book under review, and most proofs have been re-worked and simplified with respect to the original ones.

The book is written very carefully and it is essentially self-contained, also thanks to the technical appendices which contain all the necessary tools of concentration of measure, random walks and renewal theory. Occasionally the effort towards maximal generality conflicts with readability of the text, which however is globally very well written.

In conclusion, this is an excellent review on the subject for the mathematically oriented reader, and hopefully it will also allow theoretical physicists to access the language and the literature of the probabilists' and mathematical physicists' community working in this field.